

## Method 3: Solving Systems by Combination/Elimination

## Addition Property of Equality:

$$\begin{aligned} \text{If } a &= b \text{ and} \\ c &= d, \text{ then} \\ a + c &= b + d. \end{aligned}$$

- Process:
1. Obtain opposite coefficients for one variable.
  2. Combine equations to eliminate that variable and solve for the other.
  3. Substitute the result into either of the original equations to find the second variable.
  4. Check!

Example 1:  $\begin{cases} 4x + y = 8 & \textcircled{1} \\ 5x - y = 28 & \textcircled{2} \end{cases}$

Step 1: ✓  $\begin{cases} 4x + y = 8 & \textcircled{1} \\ 5x - y = 28 & \textcircled{2} \end{cases}$

Step 2: 
$$\begin{aligned} 9x &= 36 \\ x &= 4 \end{aligned}$$

Step 3:  $\begin{aligned} \textcircled{1} \quad 4(4) + y &= 8 \\ 16 + y &= 8 \\ y &= -8 \end{aligned}$

Step 4: Check

$$\begin{aligned} \textcircled{1} \quad 4(4) + (-8) &= 8 \\ 16 + (-8) &= 8 \checkmark \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 5(4) - (-8) &= 28 \\ 20 + 8 &= 28 \checkmark \end{aligned}$$

Solution:  $(4, -8)$

Example 2:

Step 1:

$$\begin{array}{l} \textcircled{1} \begin{cases} 4x - 3y = 15 \\ 2x - 3y = 9 \end{cases} \xrightarrow{\times(-1)} \begin{cases} 4x - 3y = 15 \\ -2x + 3y = -9 \end{cases} \\ \textcircled{2} \end{array}$$

Step 2:  $2x = 6$   
 $x = 3$

Step 3:  $\textcircled{2} \begin{cases} 2(3) - 3y = 9 \\ 6 + (-3y) = 9 \\ -3y = 3 \\ y = -1 \end{cases}$

Step 4: Check  
 $\textcircled{1} 4(3) - 3(-1) = 15$   
 $12 + 3 = 15 \checkmark$   
 $\textcircled{2} 2(3) - 3(-1) = 9$   
 $6 + 3 = 9 \checkmark$

Solution:  
 $(3, -1)$

Example 3:

Step 1:

$$\begin{array}{l} \textcircled{1} \begin{cases} 2x - 3y = -3 \\ 10x + 7y = -4 \end{cases} \xrightarrow{\times(-5)} \begin{cases} 2x - 3y = -3 \\ -10x + 15y = 15 \end{cases} \\ \textcircled{2} \end{array}$$

Step 2:  $22y = 11$   
 $y = \frac{1}{2}$

Step 3:  $\textcircled{1} \begin{cases} 2x - 3(\frac{1}{2}) = -3 \\ 2x - \frac{3}{2} = -3 \\ 2x = -\frac{3}{2} \\ x = -\frac{3}{4} \end{cases}$

Step 4: Check  
 $\textcircled{1} 2(-\frac{3}{4}) - 3(\frac{1}{2}) = -3$   
 $-\frac{3}{2} + (-\frac{3}{2}) = -3$   
 $-\frac{6}{2} = -3 \checkmark$   
 $\textcircled{2} 10(-\frac{3}{4}) + 7(\frac{1}{2}) = -4$   
 $-\frac{15}{2} + \frac{7}{2} = -4$   
 $-\frac{8}{2} = -4 \checkmark$

Solution:  
 $(-\frac{3}{4}, \frac{1}{2})$

You Try: SNB - Solve the systems below using any method.

$$1. \begin{cases} x + 2y = 2 \\ -x + y = -11 \end{cases}$$

$$2. \begin{cases} -18x + 6y = 24 \\ 3x - y = -2 \end{cases}$$

$$3. \begin{cases} -16x + 2y = -2 \\ y = 8x - 1 \end{cases}$$

$$1. \begin{cases} x + 2y = 2 & \textcircled{1} \\ -x + y = -11 & \textcircled{2} \end{cases}$$

step 1  
✓

$$\text{step 2: } \begin{aligned} 3y &= -9 \\ y &= -3 \end{aligned}$$

$$\text{step 3: } \begin{aligned} \textcircled{1} \quad x + 2(-3) &= 2 \\ x + (-6) &= 2 \\ x &= 8 \end{aligned}$$

$$\text{step 4: check: } \begin{aligned} \textcircled{1} \quad 8 + 2(-3) &= 2 \\ 8 + (-6) &= 2 \quad \checkmark \\ \textcircled{2} \quad -8 + (-3) &= -11 \quad \checkmark \end{aligned}$$

Solution:  
 $(8, -3)$

$$2. \begin{cases} -18x + 6y = 24 & \textcircled{1} \\ 3x - y = -2 & \textcircled{2} \end{cases} \xrightarrow{\text{Step 1: } \times 6} \begin{cases} -18x + 6y = 24 \\ 18x - 6y = -12 \end{cases}$$

Step 2:

$$0 = 12$$

Always False

No solution

parallel  
lines

$$3. \begin{cases} -16x + 2y = -2 & \textcircled{1} \\ y = 8x - 1 & \textcircled{2} \end{cases}$$

Step 1: ✓

$$\text{Step 2: } \textcircled{1} -16x + 2(8x - 1) = -2$$

$$-16x + 16x + (-2) = -2$$

$$-2 = -2$$

Always true

All solutions on  $y = 8x - 1$ .